



Optimising production using the state-contingent approach versus the EV approach

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Optimising Production using the State- Contingent Approach versus the EV Approach

Institute of Food and Resource Economics (FOI)

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Optimising Production using the State-Contingent Approach versus the EV Approach

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Abstract

It is not clear whether the state-contingent approach to decision making under risk and uncertainty has the potential of providing better decisions than the well-known EV model based on an estimated stochastic production function and variance measures. The paper uses Monte Carlo simulation to analyse this question. Based on an artificially generated set of stochastic production data, parameters of both stochastic production functions and of state-contingent production functions are estimated. Using these estimated production functions, input decisions are afterwards optimised using three different types of utility functions, i.e. linear utility function, EV-utility function and a Cobb Douglas utility function based on state-contingent income measures. Monte Carlo simulation is carried out for various numbers of observations. Finally the results are compared to the true optimal choice of input, to identify the most efficient model; the EV-model or the model based on the state-contingent approach. The main result is that under certain conditions, the state-contingent approach performs better than the EV model.

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Preface

This paper was presented by Svend Rasmussen at the 375th NJF-seminar: “Farm Risk Management in Oslo, Norway, June 2006.

Mogens Lund

1. Introduction

The classical approach to the problem of optimizing production under uncertainty is the Expected Utility (EU) model. The EU model is, in its basic form, relatively general. However, the tradition has developed over time that the EU model is applied empirically as a model where utility as a function of the expected value and variance of profit is maximized (EV model), based on stochastic production functions (Dillon and Anderson, 1990; Hardaker et al., 2004; Robison and Barry, 1987).

Chambers and Quiggin (2000) has provided an alternative theoretical approach (the state-contingent approach) to describing and analysing production under uncertainty. Its analytical power has been demonstrated by Chambers and Quiggin in a number of subsequent papers (Chambers and Quiggin, 2002b; Chambers and Quiggin, 2002a; Chambers and Quiggin, 2001; Quiggin, 2002; Quiggin and Chambers, 2001; Quiggin and Chambers, 2004), and by Rasmussen (Rasmussen, 2003; Rasmussen, 2004) by using it to derive criteria for optimal production under uncertainty applying well known marginality principles.

One of the main conclusions in both Rasmussen (2004) and Rasmussen (2006) is that while the state-contingent approach has clear analytical advantages, it is not clear whether it has any empirical advantages for normative (prescriptive) purposes. The mere fact that application of the model presupposes knowledge (estimation) of each individual state-contingent production function certainly limits one's expectations in this regard. It is unrealistic to expect that production functions may be estimated for all possible states of nature. Indeed, state-contingent production functions may be estimated for just a few. The only attempt to estimate state-contingent production functions known to the author is Griffith and O'Donnell (2004), who estimated production functions for three separate states of nature.

Also, Just (2003) expresses serious doubts about the advantages of the state-contingent approach, compared to the parametric distributional representation of risk used in, for example the EV model. He states that most distributions facing farmers have a large number of potential outcomes (states of nature), and that the relative advantages of the state-contingent approach versus the classical EU approach depends on the number of states of nature relative to the number of distributional moments required for adequate representation of the problem (Just, 2003, p. 140).

Rasmussen (2006) suggested that the question of which approach is best - the classical approach based on the EU model or the state-contingent approach - can be tested by Monte-Carlo simulation. The objective of the present paper is to carry out such a test.

2. The problem – general formulation

Empirical application of the state-contingent approach in a decision-making context is not just a matter of estimating state-contingent production functions. It is also a question of whether the state-contingent approach has the potential for providing a better framework for decision-making, than the traditional Expected Utility (EU) approach.

As mentioned above, the EU approach is a relatively general approach. In fact, the EU model does *not* presuppose any specific functional form of the technology, nor of the probability distribution of the stochastic outcomes. All it presupposes is that the utility function is the expected value of utility measures estimated using a von Neumann-Morgenstern utility function.¹⁾ Conversely, the state-contingent approach does not require (nor exclude) any specific form of the utility function.

Empirical applications of the EU model in a decision-making context have taken various forms; from those based on an estimated von Neumann-Morgenstern utility function to those based on the expected value-variance utility function (EV model) and efficiency measures based on stochastic simulation using numerical probability density functions (Hardaker et al., 2004). However, when it comes to modelling stochastic production, application of the EU model has typically been based on what Chambers and Quiggin call “stochastic” (as opposed to state-contingent) production functions.

In order to compare the EU with the state-contingent approach, it is necessary to choose a specific form of the EU model. As the expected value-variance (EV) model seems to be the standard in applied work on decision-making under uncertainty, it is used in what follows as the basis for the comparison.

To compare the two approaches it is also necessary to define the basis for the comparison. In this paper the comparison of the two approaches is based on their ability to identify the true optimal production decision, *the true optimal production decision* being defined as the production decision (choice of input) that maximizes utility of a decision-maker having *complete information*.²⁾

¹ For review of the EU model see for instance (Moschini and Hennessy, 2001).

² The term “complete information” is used in the sense, that the decision-maker knows all possible states of nature, their consequences, and their relative frequencies of occurrence.

The problem dealt with in the following may now be stated as follows: With *incomplete information* (a sample of production (input-output) data from the true world), is it better to base decision-making concerning production on estimated state-contingent production functions and application of the state-contingent approach, or does the well-known EV model based on an estimated stochastic production function and variances provide just as good – or maybe even better decisions?

The following section will describe the approach in more detail.

3. Stochastic versus state-contingent production functions

Consider first *stochastic production functions*, i.e., functions of the type:

$$(1) \quad z = f(\mathbf{x}, \varepsilon)$$

where z is output, \mathbf{x} is a vector of inputs, and ε an error term.

The empirical problems concerning application of stochastic production functions are related to choosing the functional form and estimating (1) and the probability density function of the error term ε , or at least its first two moments. This approach is well-known, and has been extensively dealt with in the literature. The Just-Pope form ($f(\mathbf{x}, \varepsilon) = g(\mathbf{x}) + h(\mathbf{x})\varepsilon$) (Just and Pope, 1978) has proven especially popular in applied analyses (Moschini and Hennessy, 2001). The quantity and the quality demanded of the data to estimate a specific function depends on the method of estimation. The demand of classical Ordinary Least Squares (OLS) is well described in the econometric literature.

Next, consider the state-contingent approach based on *state-contingent production functions*:

$$(2) \quad z_s = f_s(\mathbf{x}) \quad (s = 1, \dots, S)$$

where z_s is output in state s , and $f_s(\bullet)$ is a production function specific to state s , and S is the total number of states.

The empirical problems concerning application of state-contingent production functions are related to identifying the possible S states, choosing the S functional forms of $f_s(\bullet)$, and estimating them. In the ideal case the data set includes observations of all S states. This is required (again depending on the method³) chosen for estimation) for estimation of the state-contingent production functions.

This ideal case is unlikely as the *number of possible states (S) is often very large* (Just, 2003) so that with limited data there will be potential states for which there are

³ If OLS is used there are certain demands on the number of observations (the problem of degrees of freedom DF). Alternative methods (for instance Maximum Entropy) do not involve the same demand on number of observations.

no observations available, or where there are not enough observations to provide the necessary degrees of freedom for estimation (see Rasmussen (2006) for illustration).

Secondly, a state of nature is often characterized by a large number of relevant⁴⁾ state variables. Where only some of these variables are observed/recorded in experiments that create the data, then the state-description is *incomplete*. If the data refer to crop production, the variables actually recorded could be e.g. monthly rainfall and hours of sunshine per month. However, other variables (like for instance wind velocity or CO₂ content of the atmosphere, etc.) may influence production. If these variables are not observed (and recorded), then the state description is again incomplete; the *recorded state* is an incomplete description of the real state. It is of course only possible to estimate state-contingent production functions that refer to the *recorded states*. If the recorded states are incomplete descriptions of the real states, then the estimated state-contingent production functions will be *stochastic production functions*, because the level of the non-recorded state variables may vary from one observation to the other.

The data used to estimate the production functions in the following are generated using Monte-Carlo simulation. To imitate the real world, the data were generated in a way so that that the *state description is incomplete*. Further, to analyse the problems involved when there is not enough degrees of freedom to estimate a production function for certain states of nature, data sets both with different lengths of time series and various combinations of time series and cross section data (panel data) were generated. Details are shown below.

⁴ By “relevant” state variables I mean state variables that influence output. Thus, if the CO₂ content in the atmosphere influences the output of barley, then the CO₂ content in the atmosphere is a relevant state-variable.

4. The simulation set-up

4.1. The benchmark and the method of comparison

To compare the EV and the state-contingent model it is necessary to have a benchmark. In this case the benchmark is decisions made by a fully informed decision-maker according to the following complete information case:

I. The benchmark (the true world (complete information) case):⁵⁾

1. A finite set of real states $\Omega = [1, \dots, S]$
2. The individual states occur with known frequencies (π_1, \dots, π_S)
3. The decision maker has a known utility function W in state-contingent incomes q_s :
$$W(\mathbf{q}) = W(q_1, \dots, q_S)$$
4. Production of a single output z , from known production technology:
$$z_s = f_s(\mathbf{x}) \quad (s \in \Omega)$$
5. Output price p and input prices w_i ($i = 1, \dots, n$) for n inputs given

The state-contingent income q_s is:

$$(3) \quad q_s = z_s p - \sum_{i=1}^n w_i x_i - c^F + k_s$$

where c^F is fixed costs and k_s is a state-contingent income from other sources than production.

By choosing the values of all the parameters in 1. – 5. and the functional forms of $W(\bullet)$ and $f_s(\bullet)$ for $(s \in \Omega)$, it is possible to calculate the vector \mathbf{x}^* of optimal amounts of input, i.e., the vector of input that maximizes utility W .

Next, consider the world as it is observed by the imaginary researcher doing applied work:

⁵⁾ The term “true world” is here used in the sense that this is a description of the world as it actually is: The state description is complete, all possible states are recorded, the relative occurrence of each individual state is known, the relation between input and output in any state is known (with certainty), and the true preferences of the imaginary decision maker are known. The term “complete information” is used in the sense that the decision maker knows all about the “true world” and makes decisions accordingly.

II. The observed world including the following:

1. A sample of T historical observations ($t=1, \dots, T$) of input (\mathbf{X}) and corresponding output (\mathbf{z}), where \mathbf{X} is a $(T \times n)$ matrix where the element x_{ti} is the amount of input i applied in year (observation) t , and \mathbf{z} is a $(T \times 1)$ vector of outputs z_t . The subset of observations in which a given recorded state r occurs is T_r . Thus, the total set of observations $\{1, \dots, T\}$ is $\bigcup_{r \in \Omega^R} T_r$, where Ω^R ($\Omega^R \subset \Omega$) is the set of states that has in fact been observed and recorded, but where the state description is incomplete in the sense that only *some* of the relevant state variables have been recorded⁶. Further, we define the sub-vector \mathbf{z}^r as a vector of observed outputs in state r , with elements z_t^r ($t \in T_r$).
2. Output price p and input prices w_i ($i = 1, \dots, n$) given.
3. Fixed cost c^F and other income $k_s = \bar{k}$ given.

The question posed earlier in Section 2 can now be restated as:

With the objective of assisting the decision maker to maximize utility, what is the most efficient way to use this sample of the T historical observations:

1. To estimate a stochastic production function $\hat{z} = \hat{f}(\mathbf{x})$ on which to base the decision in a EV model? or
2. to estimate state-contingent production functions $\hat{z}_r = \hat{f}_r(\mathbf{x})$, ($r \in \Omega^R$) and to base the decision on these?

In the first case, the implicit assumption is that the decision maker optimizes production according to the following EV utility function:

$$(4) \quad W = W_1(\hat{f}(\mathbf{x}), \hat{\varepsilon}_1, \dots, \hat{\varepsilon}_T, \boldsymbol{\alpha}) = W_1(E, V, \lambda)$$

where $\boldsymbol{\alpha}$ is a vector of constants ($p, w_1, \dots, w_n, \lambda, c^F, \bar{k}$) of which λ is the coefficient of absolute risk aversion (see below), $\hat{\varepsilon}_t$ are the residuals:

$$\hat{\varepsilon}_t = z_t - \hat{z} \quad (t = 1, \dots, T),$$

⁶ In the following, the indices R and r refer to Recorded/ recorded states while the indices s and S refer to real states.

and E and V are the expected net income $\hat{z}p - \sum_{i=1}^n w_i x_i - c^F + \bar{k}$ and the variance of the net income $V(z_i p - \sum_{i=1}^n w_i x_i - c^F + \bar{k})$, respectively.

In the second case the implicit assumption is that the decision maker optimizes production according to the following utility function:

$$(5) \quad W = W_2(\hat{f}_1(\mathbf{x}), \dots, \hat{f}_R(\mathbf{x}), \mathbf{a})$$

where \mathbf{a} is the same vector of constants as above.

In (4), the uncertainty is quantified in the form of the estimation residuals. In (5) the uncertainty is quantified by the R state-contingent production functions and the relative frequency by which the individual states occur (explicitly or implicitly modelled as part of the utility function W_2).⁷

4.2. The true world (the stochastic production environment).

To perform the comparison as described above it is necessary to control the benchmark. If one does not know the true optimal decision, then it is not possible to measure which of the two approaches (the EV approach or the state-contingent approach) performs the best, in the sense of “coming closest to the true optimal decision”. Therefore, it is necessary to fully control the data generating process and to define the preferences of the (imaginary) decision maker.

To keep things simple but still realistic, a Cobb-Douglas functional form with one output and three inputs was used to generate the production data. Thus the production function:

$$(6) \quad z_s = A_s x_1^{a_{1s}} x_2^{a_{2s}} x_3^{a_{3s}} \quad (s = 1, \dots, S),$$

⁷ Estimation residuals may also be included in the utility function in the second case. In that case, the utility function takes the form $W = W_2(\hat{f}_1(\mathbf{x}), \dots, \hat{f}_R(\mathbf{x}), \hat{\varepsilon}_1, \dots, \hat{\varepsilon}_R, \mathbf{a})$ where the residuals $\hat{\varepsilon}_r$ are now $\hat{\varepsilon}_r = \hat{z}_r - E(\hat{z})$ ($r \in \Omega^R$), and where $E(\hat{z}) = \sum_{r=1}^R \pi_r \hat{z}_r$ (the index r to refers to a recorded state). This form of the utility function is also considered (see below).

where z_s is the output in state s , x_1 , x_2 , and x_3 are three variable inputs, and A_s , a_{1s} , a_{2s} , and a_{3s} are the parameters of the Cobb-Douglas production function in state s , was used to generate the “true” relationship between input and output for a given state (s) of nature.

The various real states of nature were generated by combining the following values of the four parameters of the Cobb-Douglas production function.

Using all possible combinations of these parameter values of the four parameters, it is possible to generate a total of $3 \times 4 \times 4 \times 4 = 192$ real states of nature ($S = 192$), and a corresponding number of state-contingent production functions. For the purpose of this paper, these 192 states describes the “true world”.

The choice of parameter values in table 1 is more or less arbitrary. The primary concern was to model certain variability between different states of nature; at the same time maintaining customary assumptions concerning production technology, i.e., decreasing marginal returns of the individual inputs, and decreasing returns to scale.⁸⁾

Table 1. Values of parameters (“state variables”)			
A	a_1	a_2	a_3
2	0.05	0.12	0.24
4	0.12	0.22	0.28
5	0.19	0.32	0.32
	0.26	0.42	0.36

The stochastic environment was generated using the relative frequencies shown in table 2.

⁸ This last point proved to be crucial. On the one hand, if prices are given (competitive market) it is not possible to identify a profit maximizing input, if returns to scale is not (eventually) decreasing. On the other hand, one should not exclude the possibility that at the profit maximizing level of input, returns to scale may be increasing in certain states of nature, as exemplified by the combination $a_1=0.26$, $a_2=0.42$, $a_3=0.36$ in Table 1. The choice of including this one state in which returns to scale is increasing, caused problems when optimizing production as shown later.

Table 2. Relative frequencies of parameters in table 1

A	a_1	a_2	a_3
0.20	0.10	0.15	0.20
0.50	0.35	0.30	0.30
0.30	0.45	0.40	0.30
	0.10	0.15	0.20

These frequencies are (arbitrary) relative numbers describing the probability of occurrence of each of the four “state variables” (the sum of probabilities in each column is 1). For instance, the probability that the parameter a_2 takes the value 0.32 is 0.40. To illustrate, the probability of observing a state of nature characterized by $A=4$, $a_1=0.19$, $a_2=0.12$, and $a_3=0.36$ is $0.50 \times 0.45 \times 0.15 \times 0.20 = 0.00675$.⁹ The values of the relative frequencies were chosen, so that some states are more likely than others.

Application of input was generated using the following table of possible amounts of input (table 3).

Table 3. Application of input (“experiment plan”)

x_1	x_2	x_3
40	90	55
55	120	75
70	150	100
85	180	125
100	210	150
115	240	165

Using all possible combinations of input it is possible to generate $6 \times 6 \times 6 = 216$ production plans.

The choice of amounts of input in Table 3 is also arbitrary. The primary concern was to provide enough variation that subsequent problems estimating production functions would not be due to lack of variation in of input while avoiding multicollinearity.

⁹ In real life the state variables will typically *not* be independent. Thus, if the relevant state variables are for instance sunshine, rain, temperature, and wind velocity, these four variables are typically not independent variables. However, in the context considered in this paper, this is not important, and the individual state variables are for convenience considered as being independent.

Also the level of input should cover the utility maximizing part of the production function.¹⁰

Concerning the application of the input in table 3, imagine to ease understanding, that the production data come from an (imaginary) experiment station. This experiment station has a simple “experiment plan” which is to apply each year an amount of the three inputs x_1 , x_2 , and x_3 , determined by randomly drawing an amount of input from the individual columns in Table 3. Thus, in any year, the experiment station may have applied a combination of for instance 55 units of x_1 , 180 units of x_2 , and 125 units of x_3 . The specific combination of these amounts of input occurs in the data set with a relative frequency of $1/6 \times 1/6 \times 1/6 = 1/216$.

To facilitate analysis of the advantage of having available *panel data*, assume further that the experiment station has not only one, but several *plots of land*, on which it performs trials concerning application of the three inputs x_1 , x_2 , and x_3 . Thus each year, the experiment station has one or more *plots of land* on which it carries out experiments as just described. The different plots are treated independently in the sense that the amount of each of the three inputs is determined independently for each plot. The quality of land, the macro- and micro climate, the quality of management (or more generally, *the real state*) is the same on each plot within the same year. The only reason that the amount of output may vary from one plot to another is therefore different amounts of input.

Combining the 192 possible real states of nature and the 216 possible production plans, provides a potential of simulating $192 \times 216 = 41,472$ different amounts of output.

4.3. Monte Carlo simulation

The stage is now set for generating production data to be used for estimating state-contingent and stochastic production functions, i.e., to generate a sample of T observations as referred to in Section 4.1.

The two methods to be compared are to either:

¹⁰ A number of pre-tests were performed to identify the relevant level of each input.

1. use the sample of T observations of outcome from the true world to estimate one stochastic production function $\hat{f}(\mathbf{x})$, and base decision-making on this production function as in (4)

or

2. use the sample of T observations of outcome from the true world to estimate R state-contingent production functions $\hat{f}_s(\mathbf{x})$ ($s \in \mathcal{Q}^R$), and base decision-making on these R production functions as in (5).

The procedure of using the first method is well-known and (relatively) straight forward. The procedure of using the second method implies the choice of the number R , i.e., the number of state-contingent production functions to estimate. This in turn is determined by how many states that are actually *recorded*.

As mentioned above, it is unrealistic to assume that observations for all (192) states will be available. To make a realistic¹¹⁾ comparison, one should choose the number R as the number of states that would typically be recorded in cases where the total number of (real) states is 192. But how many (relevant) states of nature are there? And how many of these states are typically recorded by experiment stations when making experiments?

More specifically: If the state of nature (s) is the physical climate (measured by a number of variables (state variables)), how many different types of climate to affect the yield of wheat? Further: How many of these real states are typically recorded by experiment stations (or other relevant institutions, for instance regional meteorological institutes)?

Answering these questions might be a research project in itself. Instead of trying to amplify these questions, the assumption made here is that the number of possible states of nature is large, and that the number of recorded states is comparatively small. What is “large” and “small” in this context will not be discussed further. The actual choice made here is that compared to the total number of 192 real states used in this paper, the number of recorded states is 12.¹²⁾

¹¹⁾ Realistic from a (potential) real world application point of view.

¹²⁾ This would correspond to for instance 4 levels of rain combined with 3 levels of temperature during the growing season.

The recorded states are modelled by assuming that the level of the two state variables A and a_1 is recorded by the experiment station. Therefore it is possible to identify the value of A and a_1 for each observation (year). On the other hand, the value of a_2 and a_3 are not recorded, and therefore may assume any of the four values specified in Table 1 with probabilities specified in table 2.

Based on the assumption just mentioned, the *data generating process* now runs as follows:

For each year t ($t=1 \dots T$), the following steps are carried out:

1. The amount of input applied to a “plot” is determined by random choice of the possible input amounts of x_1 , x_2 , and x_3 in Table 3. These amounts of input applied to the plot in question are recorded.
2. In the case of more plots per year, the procedure in 1) is repeated for every plot.
3. The state of nature in the year in question is determined by drawing individually the four state variables in Table 1 randomly, according to the probabilities in table 2.¹³
4. If the data are to be used to estimate the 12 state-contingent production functions, *the value of two state variables 1 and 2 (i.e., the value of parameter A and a_1) are recorded* (the values of state variables 3 and 4 (here a_2 and a_3) are *not* recorded). If the data are to be used to estimate a single stochastic production function, *none of the four state variables are recorded*.
5. The amount of output y is calculated for each plot using the production function (6) by inserting the relevant amounts of input determined in step 1) (and step 2)), and the parameter values determined in step 3). The amount of output z is recorded (for each plot).
6. The year number t is recorded

To analyse the consequence of having available different time series and a different number of plots per year, simulation was carried out for different numbers of years and plots.

¹³ The RANUNI random number generator in SAS was used to generate random numbers.

5. Estimation of state-contingent production functions

As described in Section 4, simulation of production data was carried out by drawing data from a stochastic production environment (Ω) including 192 different real states of nature. To imitate the typical situation faced by the researcher when carrying out applied research, the choice was made to record only two of the four stochastic state variables, rendering it possible to estimate a maximum of only 12 state-contingent production functions.

In applied work, one has to consider the choice of functional form. In the present simulation case it is known that the data come from a Cobb-Douglas production technology (see Section 4.2). Therefore, the obvious choice of functional form of the 12 state-contingent production functions is also Cobb-Douglas.¹⁴ The production functions to be estimated are therefore the following:

$$(7) \quad z_r = B_r x_1^{b_{1r}} x_2^{b_{2r}} x_3^{b_{3r}} \quad (r = 1, \dots, 12)$$

where r is an index of recorded state of nature, and B_r , b_{1r} , b_{2r} , and b_{3r} are parameters.

5.1. The econometric model

The econometric model corresponding to the production function in (7) is:

$$(8) \quad \ln z_{\tau,r} = \ln B_r + b_{1r} \ln x_{\tau,1} + b_{2r} \ln x_{\tau,2} + b_{3r} \ln x_{\tau,3} + \varepsilon_\tau$$

where ε_τ is the error term with an expected value zero, and variance σ_ε^2 .¹⁵

5.2. Data samples

To analyse the consequence of having available different sample sizes, estimations were carried out for simulated samples including 50 years, 100 years, 200 years, and 400 years, respectively. At the same time, the number of plots was varied from one to

¹⁴ In applied work without prior knowledge of the technology which generated the data, the researcher would probably choose a more flexible functional form.

¹⁵ For interpretation of the two parameters b_{2r} and b_{3r} , see Appendix 2.

three plots, so as to measure the consequence of having available more observations for the same state of nature.

With more than one plot, the model (8) changes to:

$$(9) \quad \ln z_{\tau,r,d} = \ln B_r + b_{1r} \ln x_{\tau,1,d} + b_{2r} \ln x_{\tau,2,d} + b_{3r} \ln x_{\tau,3,d} + \varepsilon_{\tau,d}$$

$$(r=1,\dots, 12; \tau \in T_r; d=1, 2, 3)$$

where d is an index of plot.

5.3. Estimation of parameters.

As the econometric model (9) is linear in the parameters, Ordinary Least Squares (OLS) can be applied directly to estimate each of the 12 state-contingent production functions.

It is easy to show that the error term in ε_τ in (8) (and (9)) is heteroscedastic, i.e., its variance increases with the amount of input (see Appendix 2). Therefore, estimation efficiency has the potential of improving, if Weighted Least Squares is applied (Judge et al., 1982, p. 409).

Estimation was performed using both methods, i.e., both Weighted Least Squares and Ordinary Least Squares. Generally, the estimated parameters were almost the same using the two methods. However, the precision was higher (the standard deviations of the estimate were lower) when using Weighted Least Squares (compare table A1 and table A2 (Appendix 1)).

To compare the effect of having data with more plots per year, estimation was performed for one, two and three plots per year, respectively. Along with the assumption of having 50 years, 100 years, 200 years, and 400 years of data available, respectively, a total of $3 \times 4 = 12$ estimations of each of the 12 state-contingent production functions were performed.

The assumption is made that the soil quality, the management, the technology, and the state of nature are the same across plots of land within the year. The only thing that varies between plots within years is therefore the amount of the three inputs, x_1 , x_2 , and x_3 . Therefore, the restriction was applied, that parameters are identical across

plots. This restriction implies simulation of the extreme case of panel data quality. It is therefore possible to test the value of having available panel data of the highest quality when estimating state-contingent production functions. This is done by considering the regression equations for each plot, and estimating the regression equations simultaneously with the restriction that the parameters are equal across plots.

All estimations were carried out by using the procedure PROC SYSLIN in SAS 8.02 applying the estimation method ITSUR, when there were two or three plots.

5.4. Results of parameter estimation.

In a number of cases there were not enough observations to estimate a state-contingent production function for certain states. This was (of course) especially the problem with a “short” time series (50 and 100 years), but even with 200 year series there were sometimes not enough observations of certain (rare) states to have enough observations (more than 5 observations) to perform estimation. Only with 400 years there were always enough observations within each state to perform estimation.

Two hundred simulations were carried out. The estimated parameters are not reported here, but are available upon request. However, to illustrate, Table A1 and A2 show a sample of results concerning the state-contingent production function of state 2 ($s_{1,2}$). With datasets of only 100 years of observations, it was only possible to estimate production functions in 135 of the simulation cases. In the other 65 simulation cases there were not enough observations of state 2. Therefore, the results concerning 100 year series only cover 135 simulation runs. With 200 years of observation, it was possible to estimate parameters in all the 200 simulation runs.

When comparing the results of one, two, and three plots (see table A1 and A2 in Appendix 1), it is interesting to notice the extreme improvement in estimation efficiency achieved when going from one to two plots per year. In many cases the error (the standard deviation) of the estimated parameters is reduced by a factor 10 or more. Going from two to three plots only improves the estimation efficiency marginally. This result is not in itself surprising, and it underlines the fact that when estimating state-contingent production functions, it is of greater importance to have a number of observations that are known to be observations from the same *real* state of nature (i.e., observations *within years*), than to have observations from the same *recorded states* (i.e., observations *over years*).

The precision of estimation increases (the standard deviation decreases) with the number of observations in the sample, as expected.

The results in Table A1 and A2 show that when the time series is 200 years and more and the number of plots is two or more then the average (average over two hundred simulations) estimate of the two parameters B and b_1 , are pretty close to the true parameter value (A and a_1 , respectively (see Table 1)). The average value of the estimate of the two other parameters b_2 and b_3 are pretty close to \bar{a}_2 and \bar{a}_3 , respectively. As described in Appendix 2, this is not a coincidence.¹⁶

¹⁶ Parameters for all 12 state-contingent production functions are available from the author upon request.

6. Estimation of stochastic production functions

The classical approach (the non-state-contingent approach) to estimation of production technology under uncertainty is estimation of stochastic production functions. Since Just and Pope published their paper in 1978 on comparison of alternative functional forms (alternative ways of specifying the error term), the popular choice of stochastic production function has been the Pope-Just production function:

$$(10) \quad z_t = g(\mathbf{x}) + h(\mathbf{x})\varepsilon_t$$

where $g(\mathbf{x})$ and $h(\mathbf{x})$ are log-linear forms, i.e., popular forms as either Cobb-Douglas or Translog functional forms.

The parameters of (10) may be estimated using nonlinear, stepwise estimation as described in Just and Pope (Just and Pope, 1979). In the present case, the technology is known to be Cobb-Douglas. It is therefore appropriate to choose the Cobb-Douglas as the functional form of the stochastic production function, or rather the log-linear form of the Cobb-Douglas production function, as the function $g(\bullet)$ in (10), i.e.:

$$(11) \quad z = Cx_1^{c_1}x_2^{c_2}x_3^{c_3}$$

where C , c_1 , c_2 and c_3 are the parameters.

6.1. Econometric model

Concerning the error term $h(\mathbf{x})\varepsilon_t$, it is relatively easy to show that the error term of the log-form (the estimation version) of (11) is heteroscedastic (see Appendix 3). The econometric version of model (11) is:¹⁷

$$(12) \quad \ln z_\tau = \ln C + c_1 \ln x_{1,\tau} + c_2 \ln x_{2,\tau} + c_3 \ln x_{3,\tau} + \varepsilon_\tau \quad (\tau=1, \dots, T).$$

¹⁷ For interpretation of the parameters, see Appendix 3.

6.2. Data samples

Data was generated as described in Section 4.3. Estimations of the stochastic production functions in (12) and variances were carried out for simulated samples including series of 50 years, 100 years, 200 years, and 400 years, respectively. The number of plots was varied from one to three plots to measure the consequence of having available more observations for the same state of nature.

6.3. Estimation of parameters.

As the econometric model in (12) is linear in the parameters, OLS can be applied directly. However, as the variance of the error term is not constant (depends on the input vector \mathbf{x} (see (29) in Appendix 3)), estimation efficiency may be improved by using Weighted Least Squares.

With *more than one plot per year*, it is assumed that the parameters are identical across plots, i.e., that the soil quality, the management, the technology, and the state of nature is the same on every plot within the year. Therefore, the same restrictions were applied as mentioned in Section 5.3. As before, estimation was performed using the iteratively seemingly unrelated regression (ITSUR), which is readily available in SYSLIN in SAS.

As the EV utility function is based on *expected value* and *variance of income*, we need to estimate the parameters of the stochastic production function, as well as the variance of the yield. As the estimation of the production function parameters are based on logarithmic values of input and output, and as the variance is heteroscedastic (see above), the variance does not emerge directly from an estimation of the production function. Estimation of the variance is described in Appendix 4.

6.4. Results of parameter estimation.

Averages of estimated parameters of the stochastic production function for 200 simulation runs are shown in Table A3 (OLS) and A4 (Weighted Least Squares) (Appendix 5).

Comparing the two tables in the Appendix shows that the precision increases when using weighted least squares.

The precision of estimation increases (the standard deviation decreases) with the number of observations in the sample as expected. As with the state-contingent production functions (see Section 5), it is striking how the precision of the estimation increases by increasing the *number of plots*. By merely going from one plot to two, the standard deviation of the parameter estimates typically decreases by a factor of 10. The additional increase in precision by going from two plots to three is, however, relatively modest. These results show the potential maximum gains in precision when panel data are available.

Having estimated the state-contingent production functions in Section 5 and the stochastic production function in Section 6, we are now ready to compare the two approaches as a basis for decision-making. However, first the benchmark on which to base the comparison will be described in Section 7.

7. Optimal production: The benchmark

The benchmark for comparison of the state-contingent approach and the EV model is the optimal production choices taken by a decision maker having complete information. A decision maker with complete information is here defined as a decision maker who has knowledge of the “true world” corresponding to “the benchmark” described in the beginning of Section 4.1.

To describe decision makers’ (true) preferences, three alternative forms of the utility function

$W(q_1, \dots, q_S)$ in Section 4.1 were applied:

- 1a: Linear utility function
- 1b: Cobb-Douglas utility function
- 1c: E-V utility function

The linear utility function is defined as:

$$(13) \quad W = \sum_{s=1}^S q_s \pi_s$$

The Cobb-Douglas utility function is defined as:

$$(14) \quad W = \prod_{s=1}^S q_s^{\pi_s}$$

where $0 < \pi_s < 1$ ensures that W is quasi-concave and the decision maker therefore risk averse¹⁸.

The EV utility function is:

¹⁸ The coefficients of the Cobb-Douglas function must be proportional to the probabilities of the individual states of nature, i.e., $b_i = h\pi_i$ ($i = 1, \dots, 192$) where h is some constant ($h > 0$; $h \times \max\{\pi_1, \dots, \pi_S\} < 1$) (see Rasmussen (2006)). (It turned out that the value of h did not influence the solution, so the value $h = 1$ was chosen).

$$(15) \quad W = \sum_{s=1}^S q_s \pi_s - \frac{1}{2} \lambda \sum_{s=1}^S \pi_s (q_s - E(q))^2$$

where the expectation is taken over the 192 real states, and where λ is the Arrow-Pratt coefficient of absolute risk aversion ($\lambda > 0$ to ensure risk aversion).

The linear utility function and the EV utility function are well known from the literature. The Cobb-Douglas utility function and its properties are described in more detail in Rasmussen (2006).¹⁹

The state contingent income q_s ($s=1, \dots, 192$) is given in (3) and repeated here for convenience:

$$(3) \quad q_s = z_s p - \sum_{i=1}^n w_i x_i - c^F + k_s$$

The following parameter values were applied:

- $-c^F + k_s = 5,000$ ($s = 1, \dots, S$). The level of fixed income (initial wealth) is set at an arbitrary level of 5,000.
- A, a_1, a_2, a_3 , and a_4 . The set of possible values of the production functions given in table 1.
- π_1, \dots, π_{192} . The probability of each individual state is estimated as: $p(A) * p(a_1) * p(a_2) * p(a_3)$, where $p(\bullet)$ are the probabilities of the individual state variables shown in table 2.

¹⁹ The Cobb-Douglas utility function features constant relative risk aversion (CRRA) (the expansion path is a straight line through origin) and therefore decreasing absolute risk aversion (DARA). The Cobb-Douglas function differs from the EU model in the sense that the marginal utility of income in state s depends not only on the relative probability of state s and of the net return in state s , but also on the net return in the other states of nature. In this sense, even the relatively simple Cobb-Douglas function potentially provides more flexibility in the description of preferences than that based on the popular Neumann-Morgenstern utility functions (see further in Rasmussen (2006)).

- w_1, w_2, w_3 . The input prices are arbitrarily set at $w_1=5, w_2=3, w_3=7$.²⁰
- p . The output prices arbitrarily set at $p=25$.

Concerning the risk aversion parameter λ in model (15), parametric analyses were performed to identify the value that provided the same behaviour (choice of input) as the Cobb-Douglas utility function. The result was $\lambda = 0.0001$. According to Hardaker et al. (1997, p.97) the coefficient of relative risk aversion (λ) is in the range 0.5–4 for risk averse decision makers. With an initial wealth of 5,000, a coefficient of absolute risk aversion λ with the value 0.0001 (as identified above) corresponds to a relative risk aversion of 0.5. Thus the Cobb-Douglas utility function applied in model 1b corresponds to a relative risk aversion of 0.5, which according to Hardaker et al. is a relatively weak degree of risk aversion.

Using these parameter values just described, the utility functions ((13), (14), and (15)) yielded the following (table 4) optimal amounts of the three inputs²¹:

Utility function	x_1	x_2	x_3	Utility (W)
Linear	174.5	307.8	220.5	7,444
Cobb-Douglas	100.9	206.8	132.8	6,644
EV model	100.6	206.8	133.6	6,710

The results in Table 4 show that the linear utility function provides the highest amount of optimal input, as expected. It is striking, however that even with a relatively weak degree of risk aversion the optimal amount of input is reduced to about 2/3 of the optimal amounts applied by a risk neutral decision maker.

The results in Table 4 are used in the following as a benchmark when comparing the state-contingent approach and the EV model.

²⁰ When the production technology has constant or increasing returns to scale, there is no optimal amount of input when prices are constant. According to Table 1, there are states featuring increasing returns to scale (e.g. $a_1=0.26; a_2=0.42; a_3=0.36$), and where there would therefore be no optimal solution. It turned out that even when the returns to scale was close to (but less than) one, the solver CONOPT in GAMS had serious problems identifying an optimal solution. The problem was “solved” by letting the input prices be a function of the amount of input applied. The following price functions were arbitrarily chosen: $w_1=4+\exp(0.003x_1)$; $w_2=2+\exp(0.003x_2)$; $w_3=6+\exp(0.003x_3)$.

²¹ The optimization procedure was performed using the solver CONOPT in GAMS.

8. Optimizing production: Comparison of the two methods

With the primary objective of comparing the classical approach (the EV model) and the state-contingent approach in a decision-making context, simulation of input optimization was carried out using the 200 simulated set of production functions estimated in Sections 5 and 6.

The two approaches (The *state-contingent approach* and the *EV approach*) are compared to each other and to *the benchmark*. Thus, there are a total of three cases to compare:

1. *The benchmark*, i.e., optimisation based on *complete information* about all 192 states of nature, i.e., full knowledge of the true values of the parameters of the 192 state-contingent production functions as shown in Table 1 and 2 (i.e., the benchmark is already described in Section 7).
2. *The state-contingent approach*, with optimisation based on estimated, state-contingent production functions for a *limited number of recorded states*, in this case based on 12 estimated state-contingent production functions (estimated as described in Chapter 5).
3. *The EV approach*, where optimisation is based on one (*average*) *production function and a measure of variance*, i.e., based on one estimated *stochastic production function* (estimated as described in Chapter 6).

The EV approach can be further sub-divided according to the value of the coefficient of absolute risk aversion λ . If $\lambda > 0$ then the decision maker is risk averse. If $\lambda = 0$ the decision maker is risk neutral, and the *utility function is linear* in expected net income.

To analyse more generally the consequence of choosing different types of utility function it was decided to apply the same three types of utility function as used in the benchmark case to all three cases. Thus the following three types of utility functions were applied²²:

- a. Linear utility function (risk-neutral decision maker)
- b. Cobb-Douglas utility function (risk-averse decision maker)

²² The Cobb-Douglas utility function cannot be used in case 3. Therefore only the linear and the EV utility function were applied in this case.

- c. EV-utility function (risk-averse decision maker)

8.1. The comparison plan

The full plan for comparing the different approaches can be illustrated as shown in the following table 5.

Table 5. Plan for comparison based on Monte Carlo simulation			
Utility function	Number of recorded states		
	192 Complete information	12 Estimated parameters	1 Estimated parameters
Linear	1A	2A	3A
Cobb-Douglas	1B	2B	-
EV model	1C	2C	3C

The utility function (W) in the three cases 1A, 1B and 1C have already been described in Section 7.

The utility functions in the three cases 2A, 2B, and 2C are defined as follows:

Case 2A:

$$(16) \quad W = E(q) = \sum_{r=1}^{12} \pi_r q_r$$

$$(17) \quad q_r = pf_r(x_1, x_2, x_3) - \sum_{i=1}^n w_i x_i - c^F + k_r \quad (r=1, \dots, 12)$$

$$(18) \quad f_r(x_1, x_2, x_3) = \hat{B}_r x_1^{\hat{b}_{1r}} x_2^{\hat{b}_{2r}} x_3^{\hat{b}_{3r}} \quad (r=1, \dots, 12)$$

where the probabilities π_r ($r = 1 \dots 12$) are the probabilities of states $s_{1,1}, \dots, s_{3,4}$ calculated by multiplying the relative frequencies of state-variable 1 (A) and state-variable 2 (a_1) in Table 2. The parameters in (18) are estimates of the parameters B_r , b_{1r} , b_{2r} , and b_{3r} in the model (8) and (9) estimated in Chapter 5.

Case 2B:

$$(19) \quad W = q_1^{\pi_1} q_2^{\pi_2} \dots q_{12}^{\pi_{12}}$$

A Cobb-Douglas utility function in net incomes in 12 states of nature (risk-averse decision maker). The net income q_r ($r = 1, \dots, 12$) and all other parameters as in Case 2A.

Case 2C:

$$(20) \quad W = E(q) - \frac{1}{2} \lambda V(q)$$

$$(21) \quad V(q) = E(q^2) - (E[q])^2$$

The EV utility function in state-contingent income measures. Same value of risk aversion coefficient as in Case 1C ($\lambda = 0.0001$). The expected value of net income ($E(q)$) is calculated as in Case 2A. The expected value of squared net income ($E(q^2)$) calculated in a similar way, i.e., based on the known probabilities π_r ($r = 1 \dots 12$) which are the probabilities of states $s_{1,1}, \dots, s_{3,4}$ calculated by multiplying the relative frequencies of state-variable 1 (A) and state-variable 2 (a_1) in Table 2. All the other parameters are as for Case 2A.

The utility function in the two cases 3A and 3C are as follows:

Case 3A:

$$(22) \quad W = \hat{q} = p \hat{C} x_1^{\hat{c}_1} x_2^{\hat{c}_2} x_3^{\hat{c}_3} - \sum_{i=1}^n w_i x_i - c^F + k$$

where $\ln(\hat{C})$, \hat{c}_1 , \hat{c}_2 , and \hat{c}_3 are estimates of the parameters in model (12) (Section 6). Other parameters are as in Case 1A.

Case 3C:

$$(23) \quad W = E[q] - \frac{1}{2} \lambda \hat{V}(q)$$

where $E(q)$ is as defined in Case 3A, and $\hat{V}(q)$ is the estimated variance of q . The variance estimator is shown in (34) in Appendix 4. The other parameters are as in Case 3A.

In all cases (1A...3C), estimations of the optimal amount of input were performed for each of the 200 simulated sets of production functions covering samples of 50 years, 100 years, 200 years, and 400 years of observations, respectively, combined with one, two or three plots as described in Section 5-6.

Combining the five cases 2A, 2B, 2C, 3A, and 3C (ignoring the benchmark cases (1A, 1B, and 1C) already presented in Section 7), the four levels of number of observations (50, 100, 200, and 400 years), and the one, two, or three plots, there is a total of $5 \times 4 \times 3 = 60$ scenarios. For each of these 60 scenarios, optimal input levels were estimated for each of the 200 simulated sets of production functions mentioned in Section 5 and 6.

8.2. Plan for comparison.

In Cases 2A-2C, conditions are set as favourably as possible. Thus, the relative frequencies by which the 12 individual states occur are the true values (and not estimated values).

In Cases 1A, 1B, and 1C and 2A, 2B, and 2C, the functional forms of the utility functions are strictly based on state-contingent income measures, i.e., the utility functions in Case 1 are functions of the 192 state-contingent income measures (Case 1), or are functions of 12 state-contingent income measures (Case 2).

Concerning Case 2C, notice that the variance $V(q)$ in (21) only measures the variability *between states*. One could also have included the variance *within states*, i.e., the variance of the error terms from estimating the state-contingent production functions (estimated variance of ε_t in (8)). The implicit assumption of not doing so is that from the decision maker's point of view, this variance component is negligible compared to the variance between states. The validity of this assumption has not been tested.

In Cases 3A and 3C, the starting point is *not* state-contingent income measures, but rather the stochastic production function. In Case 3A, the decision is based on one estimated stochastic production function, and the value of the net income q . In Case 3C, the decision is (also) based on the estimated variance of this income measure, i.e., based both on q and on the variance of the error terms from parameter estimation (34) in Appendix 4.

8.3. Results: Estimated optimal amounts of input

Optimisation of the amount of input was performed using the optimiser CONOPT in GAMS.

Results from Cases 2A, 2B, and 2C are shown in tables 6a, 6b, and 6c, respectively. The results from Cases 3A and 3C are shown in table 7a and 7c, respectively. In these tables, the *Average* is the average of 200 simulation values, and the *Standard deviation* is the standard deviation estimated directly as the standard deviation of the 200 simulated estimates. *% of optimal* is the estimated average amount of input in percent of the optimal amount in the benchmark case according to the results in table 4.

As the ranking of the individual models did not depend on the estimation method (Ordinary Least Squares versus Weighted Least Squares), it was decided to show only the results of the ordinary least squares estimation. Thus, the following results are all based on parameters estimated using OLS.

Table 6a. Results; 12 state-contingent production functions. Linear utility function

Scenario	Average, units			Standard deviation			% of optimal		
	X ₁	X ₂	X ₃	X ₁	X ₂	X ₃	X ₁	X ₂	X ₃
Case-years-plots									
2a - 50 - 1	160	182	122	161	181	132	91	59	55
2a - 50 - 2	74	76	63	123	92	94	42	25	29
2a - 50 - 3	50	60	46	92	44	71	29	19	21
2a - 100 - 1	193	237	182	171	180	145	111	77	83
2a - 100 - 2	79	154	82	87	111	84	45	50	37
2a - 100 - 3	76	139	76	80	67	63	44	45	34
2a - 200 - 1	164	225	161	141	155	118	94	73	73
2a - 200 - 2	72	147	83	44	72	46	41	48	38
2a - 200 - 3	71	141	80	33	39	28	41	46	36
2a - 400 - 1	176	250	187	116	116	87	101	81	85
2a - 400 - 2	112	203	135	30	17	20	64	66	61
2a - 400 - 3	110	203	134	11	16	13	63	66	61

Table 6b. Results; 12 state-contingent production functions. Cobb-Douglas utility function

Scenario	Average, units			Standard deviation			% of optimal		
	X ₁	X ₂	X ₃	X ₁	X ₂	X ₃	X ₁	X ₂	X ₃
Case-years-plots									
2b - 50 - 1	87	130	76	97	121	83	86	63	58
2b - 50 - 2	39	61	38	46	54	44	39	30	29
2b - 50 - 3	34	55	35	36	30	27	34	27	26
2b - 100 - 1	129	186	110	112	112	86	127	90	83
2b - 100 - 2	56	118	56	27	36	23	55	57	42
2b - 100 - 3	55	116	59	22	32	34	54	56	44
2b - 200 - 1	91	160	91	67	74	57	90	77	69
2b - 200 - 2	59	128	67	12	23	14	58	62	51
2b - 200 - 3	60	129	70	10	19	11	60	62	52
2b - 400 - 1	108	197	128	60	62	44	107	95	97
2b - 400 - 2	89	178	113	10	14	11	88	86	85
2b - 400 - 3	89	179	113	8	12	9	88	86	85

Table 6c. Results; 12 state-contingent production functions. EV utility function

Scenario	Average, units			Standard deviation			% of optimal		
	X ₁	X ₂	X ₃	X ₁	X ₂	X ₃	X ₁	X ₂	X ₃
Case-years-plots									
2c - 50 - 1	83	120	70	97	117	82	83	58	53
2c - 50 - 2	36	59	34	42	47	26	36	29	26
2c - 50 - 3	32	55	35	29	26	37	32	26	26
2c - 100 - 1	122	181	111	109	109	86	121	88	83
2c - 100 - 2	57	121	59	22	32	21	57	58	45
2c - 100 - 3	57	122	59	20	30	15	57	59	44
2c - 200 - 1	93	165	98	69	76	59	92	80	73
2c - 200 - 2	62	132	71	13	22	12	62	64	53
2c - 200 - 3	63	133	72	11	21	12	63	64	54
2c - 400 - 1	114	205	134	61	63	47	113	99	100
2c - 400 - 2	95	186	119	8	13	10	94	90	89
2c - 400 - 3	95	186	119	8	12	9	94	90	89

Table 7a. Results; Stochastic production function. Linear utility function

Scenario	Average, units			Standard deviation			% of optimal		
	X ₁	X ₂	X ₃	X ₁	X ₂	X ₃	X ₁	X ₂	X ₃
Case-years-plots									
3a - 50 - 1	49	72	52	83	90	74	28	23	23
3a - 50 - 2	20	44	23	5	12	5	11	14	10
3a - 50 - 3	20	44	23	5	12	5	11	14	10
3a - 100 - 1	105	163	118	116	129	120	60	53	54
3a - 100 - 2	42	98	47	9	19	8	24	32	21
3a - 100 - 3	42	98	47	9	18	8	24	32	21
3a - 200 - 1	72	137	88	66	79	58	41	45	40
3a - 200 - 2	50	114	60	9	18	10	29	37	27
3a - 200 - 3	50	114	60	9	18	10	29	37	27
3a - 400 - 1	101	188	127	78	84	63	58	61	58
3a - 400 - 2	82	172	108	8	14	11	47	56	49
3a - 400 - 3	82	172	108	8	13	10	47	56	49

Table 7c. Results; Stochastic production function. EV utility function

Scenario	Average, units			Standard deviation			% of optimal		
	X ₁	X ₂	X ₃	X ₁	X ₂	X ₃	X ₁	X ₂	X ₃
Case-years-plots									
3c - 50 - 1	42	67	44	68	79	57	42	32	33
3c - 50 - 2	19	42	22	4	11	5	19	20	16
3c - 50 - 3	19	42	22	4	11	5	19	20	16
3c - 100 - 1	77	128	82	80	99	81	76	62	61
3c - 100 - 2	36	85	40	7	14	6	36	41	30
3c - 100 - 3	35	84	40	7	13	6	35	41	30
3c - 200 - 1	60	118	75	51	65	44	60	57	56
3c - 200 - 2	43	100	52	6	13	7	43	48	39
3c - 200 - 3	43	99	52	6	13	7	43	48	39
3c - 400 - 1	73	144	93	52	58	37	72	69	70
3c - 400 - 2	62	135	82	5	10	7	62	65	62
3c - 400 - 3	63	136	83	5	10	7	62	66	62

8.4. Results, comments and explanations

The results presented in tables 6 and 7 include several dimensions. To provide some structure, the following comments commence at the general level, becoming more specific and ending with the comments concerning the *main question of this paper: Is the state-contingent approach based on estimated state-contingent production functions better as a foundation for decision-making than the classical EV model?*

When commenting on the results in the following, terms are used as follows:

- The term “the optimal level of input” means the average of all 200 simulations.
- The term “the number of years” means the number of years on which the production function parameter estimates have been based (the length of the time series).
- The term “the number of plots” means the number of plots on which the parameter estimates have been based.

8.4.1. Number of plots

Comparison of the results for different number of plots shows the potential benefit of panel data.

Result 1: *The optimal level of input decreases considerably when the number of plots increases from one to two plots.* However, increasing the number of plots from two to three have no or only very modest effect on the optimal level of input (see all tables)

Explanation: With only one plot per year the estimated parameters are very uncertain, especially when the number of years is low. In a number of cases, the estimated parameters of the Cobb-Douglas production function were negative. With negative parameters, the unrestricted optimal amount of the corresponding input would in this case be as close to zero as possible. The Cobb-Douglas production is not defined for zero input, and in empirical contexts behaves unpredictably for argument values close to zero. To avoid this, the restriction was used that the amount of any of the three inputs could not be lower than five units. The consequence is that with negative parameters, the corresponding amount of estimated optimal input is too high. Therefore, when estimation precision increases (going from one to two plots - compare tables A1 and A2 in Appendix 1) and the number of negative parameter estimates are reduced, then the optimal amount of input decreases. In the following, I shall refer to this as *asymmetry bias*.

Result 2: *The precision increases (the standard deviation of estimated optimal amount of input decreases) when the number of plots increases.* The improved precision is especially noticeable when the number of plots increases from one to two when estimating the stochastic production functions (table 7).

Explanation: The improvement obtained by increasing the number of plots, demonstrates the efficiency value of panel data.

8.4.2. Number of years

Comparison of the results for different number of years shows the potential benefit of increasing the time series of data.

Result 3: *The optimal level of input tends to increase when the number of years increases (see all tables).*

Explanation: When the number of years increases, the precision of parameter estimation also increases. In particular the probability of obtaining negative parameters decreases, and asymmetry bias (see Result 1) is avoided.

Result 4: *The precision increases (the standard deviation of estimated optimal amount of input decreases) when the number of years increases (see all tables).*

Explanation: Sampling theory would lead us to predict this. However, it should be noticed that the improvement in estimation precision is much higher from doubling the number of plots than from doubling the number of years.

8.4.3. Complete versus incomplete information

Comparison of the results for complete versus incomplete (estimated production functions) information reveals the potential improvement of basing decision making on improved information about the stochastic production environment.

Result 5: *The estimated optimal level of input is lower when decisions are based on incomplete information (12 and one estimated production functions, respectively) compared to the cases, in which decisions are based on complete information (192 known production functions). (Compare the percentages on the right hand side of Tables 6 and 7, where input levels, based on estimated parameter values taken as a percentage of the optimal input based on complete information, are well below 100, except in a very few cases).*

Explanation: The properties of the simulated technology (Cobb-Douglas production function with states being defined by parameter values) imply that the state-specific optimal amount of input is convex

in states²³ (or rather convex in state variables). To illustrate, consider for instance the four values of the state variable a_3 in Table 1, and estimate the optimal amount of input for each of these four parameter values. The results will show that the *average optimal amount* of input for the two cases $a_3=0.28$ and $a_3=0.36$, respectively, is much higher than the optimal amount of input based on the average parameter value $a_3=0.32$. Therefore, the use of “average” parameter values (“average” state), which takes place both in the state-contingent case (12 “average” states) and in the classical case (one “average” (stochastic) production function) involves a reduction in the estimated optimal amount of input. This consequence is accentuated when aggregation increases (from 12 to one production function (compare table 6 and table 7)).

8.4.4. Cobb-Douglas versus EV based utility function

Comparison of results using the two utility functions provides the basis for evaluating whether the comparison of the complete information case (192 states) and the 12 state case influences the ranking of the two cases.

Result 6: *When optimization of input is based on the 12 estimated state-contingent production functions, the optimal amount of input is at the same level irrespective of whether the utility function takes the Cobb-Douglas or the EV form (Compare Tables 6a and 6b). Also, the standard deviation is at the same level*

Explanation: The absolute coefficient of risk aversion (λ) was scaled (value 0.0001) so that in the full information case (192 known states) the two types of utility functions were forced to provide the same optimal amount of input. If the same value of λ is used in the 12 state cases, then the parity between the two utility functions is apparently not affected.

²³ If states are ordered so that $f_1(\mathbf{x}) < f_2(\mathbf{x}) < \dots < f_S(\mathbf{x})$ and if $\alpha\pi_{s-1}\tilde{x}_{is-1} + (1-\alpha)\pi_{s+1}\tilde{x}_{is+1} > \alpha\pi_s\tilde{x}_{is}$ ($0 < \alpha < 1$), then the optimal amount of input is defined as being convex in states ($\tilde{x}_{is} = \arg \max [q_s(x_i)]$).

8.4.5. State-contingency versus stochastic production function - Risk Neutrality

Comparison of cases when decision makers have utility functions that are linear in uncertainty.

Result 7: *The state- contingent approach yields a higher optimal input level than does the stochastic production function approach.* (Compare tables 6a and 7a).

Explanation: The explanation is linked to that of Result 5. Due to the fact that the optimal amount of input is convex in states (see explanation under Result 5), then a higher level of aggregation (here proceeding from 12 states to one state) implies a lower level of optimal input.

Result 8: *The precision of the optimal level of input is lower when using the state-contingent model, than when using the stochastic production function model.*

Explanation: The number of degrees of freedom used when estimating each of the state-contingent production functions is lower, than when estimating one stochastic production function.

Result 9: *The optimal level of input is closer to the true optimum when decisions are based on state-contingent production functions, than when decisions are based on stochastic production functions.* This is irrespective of the number of years of data upon which the parameter estimates are based (compare Table 7a and Table 6a).

Explanation: The explanation is the same as in the interpretation of result 5 and 7. Due to the fact that the optimal amount of input is convex in states, then a higher level of aggregation (here going from 12 states to one) implies a lower level of optimal input.

8.4.6. State-contingency versus stochastic production function - Risk aversion

Comparison of cases when decision makers have non-linear utility functions.

Result 10: *The optimal level of input is higher when based on the state-contingent approach compared to the stochastic production function models* (For an equal number of years, estimated input in tables 6b and 6c are larger than inputs in table 7c). This is independent of which form of utility function is employed.

Explanation: The same explanation is offered as for Results 5, 7 and 9 (Compare tables 6b,6c and 7c).

Result 11: *The precision of the optimal levels of input is lower when using the state-contingent model compared to the stochastic production function models* (Compare tables 6b and 6c with table 7c).

Explanation: This is because estimated production functions are based on fewer observations.

Result 12: *The optimal level of input is - on average - closer to the true optimum when decisions are based on state-contingent production functions, compared to the cases, in which decisions are based on stochastic production function.* This can be seen from the fact that the numbers on the right hand side of the table are always smaller (for a given number of years) in Table 7c than in Tables 6b and 6c.

Explanation: The same explanation applies as in Results 5, 7, 9, and 11 (Compare tables 6b, 6c and table 7c)

8.5. Discussion of the results

The results related to the main question of this paper (presented in general terms at the end of Section 2 and in more exact terms in Section 4.1.) are Results 7 - 12 in Section 8.4 above. They show that the state-contingent approach performs better than the EV model in the sense that, *on average*, it comes closer to the true optimal amount of input than the EV model does - independently of the number of years in the datasets, and independently of the risk preferences of the decision maker. On the other hand, the precision of the estimated optimal levels of input is lower when using the state-contingent model compared to the stochastic production function approach (the EV model).

This result is hardly surprising. The information available with 12 state-contingent production functions is partially lost when aggregating the description of the stochastic production into one stochastic production function providing just two measures of production, i.e., the expected value (E) and the variance (V). Therefore, depending on the “quality” of the information lost in aggregation, the information based on 12 pieces of information should provide better decisions than information based on just two pieces of information. However, for a given set of data generating the individual pieces of information, one would expect that the precision of the 12 pieces is lower than the precision of the two pieces of information.

This consequence of losing information by aggregation is also demonstrated in Result 5, according to which the aggregation of 192 state-contingent (true) production func-

tions into 12 (estimated) state-contingent production functions results in biased estimates of the optimal amount of input.

The reason that the state-contingent approach leads to better decisions than does the EV approach, in the case analysed in this paper is that the optimal amount of input is asymmetric in states. In good states it would be worth applying high amounts of input, while in bad states it would pay off reducing input, but not in an amount commensurate to the change in profits. Thus, using “averages” (aggregation) results in “wrong” decisions.

Whether this would also be the case in practice (using real data), and whether this result can therefore be generalised is not clear. The topic has not been addressed in this or other papers known to the author. Therefore, no general conclusions can be drawn based on the results obtained in this paper, except that when the asymmetry is present, then the state-contingent approach has some improvement to offer to decision making.

Other results of the analysis should be noted. The results from one plot show that there is a very high degree of uncertainty attached to the estimated optimal amount of input with only one plot per year. Therefore, to base any decision on the recommended amount of input from a one plot estimation is a gamble. The following conclusions are therefore based only on the results from two and three plots.

The advantage of increasing the number of years materialises in two ways: 1) The average estimated optimal amount of input increases thereby coming closer to the true optimal amount of input, and 2) The standard deviation of the estimated optimal amount of input decreases.

In the cases of few observations, the standard deviations of the recommended amounts of input in the state-contingent cases are relatively high (compare the standard deviations in tables 6b and 6c with table 7c)). With a relatively short time series, the state-contingent approach therefore has a high risk of recommending a wrong decision. Thus, with time series of “only” 50 years of observations, examination of the simulation results showed, that recommendations would have been wrong in around 30 % of the times measured as the relative number of times the recommended (optimal) amount of input was lower than the average amount recommended using the EV model (19 units of x_1 , 42 units of x_2 and 22 units of x_3 , respectively).

9. Conclusion

The main result of this Monte-Carlo simulation experiment is that under certain conditions, the state-contingent approach performs better than the EV model. This comparison is in the sense that *on average*, the state-contingent approach comes closer to the true optimal amount of input than the EV model - independently of the number of years in the datasets, and independently of the risk preferences of the decision maker (risk neutral or risk averse). On the other hand, the precision of the estimated optimal levels of input is lower when using the state-contingent model, compared to the stochastic production function approach (the EV model).

These results are a consequence of the specific production technology employed. The Cobb-Douglas function with state-specific parameters implies that the state-specific optimal amount of input is convex in states, and therefore aggregation of production over states (the stochastic production function) always results in lower input than in cases of less or no aggregation (state-contingency).

If one assumes that stochastic production in general is convex in states as just described, then the results presented in this paper certainly underline the potential benefit of applying the state-contingent approach compared to the EV approach, based on stochastic production functions. When the consequences of uncertainty are asymmetric in the sense that the distribution of optimal amounts of input is skewed, then important information is lost when aggregating over states. Therefore, the potential value of applying the state-contingent approach (as opposed to the classical EV model based on stochastic production functions) is highest when such asymmetry exists.

The analysis has demonstrated the extremely high value of having available panel data of good quality when estimating production functions. The precision of estimation increases considerably when plot numbers or number of years is increased when estimating state-contingent production functions, as well as when estimating stochastic production functions.

The results clearly show that the precision of estimating even a modest number of state-contingent production functions is very low, even with a relatively long time series of data. This is due to the number of times a certain state is observed being low. Therefore, one should consider the balance between the number of recorded states, and the time series at hand. With short time series, there is no basis for a differenti-

ated approach to describing various states, because the estimation technique (OLS) does not provide parameter estimates with a sufficient degree of freedom.

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Appendix 1.

Table A1. Estimated parameters of state-contingent production function. Two hundred simulations. Ordinary Least Squares. State 2 ($s_{1,2}$)

Scenario	Average, units				Standard deviation				True values	
Years-plots	B	b_1	b_2	b_3	B	b_1	b_2	b_3	A	a_1
50 - 1										
50 - 2										
50 - 3										
100 - 1	9.24	0.35	0.34	0.30	18.38	0.58	0.59	0.56	2	0.12
100 - 2	2.71	0.15	0.27	0.27	5.02	0.33	0.26	0.22	2	0.12
100 - 3	2.18	0.17	0.26	0.25	1.34	0.40	0.20	0.13	2	0.12
200 - 1	9.12	0.21	0.28	0.31	18.03	0.42	0.37	0.33	2	0.12
200 - 2	2.05	0.11	0.27	0.27	0.43	0.06	0.05	0.05	2	0.12
200 - 3	2.02	0.12	0.27	0.26	0.32	0.02	0.04	0.02	2	0.12
400 - 1										
400 - 2										
400 - 3										

Table A2. Estimated parameters of state-contingent production function. Two hundred simulations, Weighted Least Squares, State 2 ($s_{1,2}$)

Scenario	Average, units				Standard deviation				True values	
Years-plots	B	b_1	b_2	b_3	B	b_1	b_2	b_3	A	a_1
50 - 1										
50 - 2										
50 - 3										
100 - 1	6.72	0.19	0.29	0.25	12.88	0.36	0.37	0.37	2	0.12
100 - 2	2.46	0.15	0.26	0.26	3.80	0.19	0.18	0.18	2	0.12
100 - 3	2.75	0.14	0.26	0.25	6.89	0.15	0.23	0.13	2	0.12
200 - 1	4.41	0.13	0.27	0.26	9.93	0.17	0.18	0.20	2	0.12
200 - 2	2.00	0.12	0.26	0.26	0.15	0.01	0.03	0.01	2	0.12
200 - 3	1.99	0.12	0.26	0.26	0.12	0.01	0.03	0.01	2	0.12
400 - 1										
400 - 2										
400 - 3										

Appendix 2

Comparing the simulation setup (see (6)) and the estimation setup (see (7)), the parameter B_r is an estimator of A_s and parameter b_{1r} is an estimator of a_{1s} . Further, the parameters b_{2r} and b_{3r} may be expressed as $b_{2r} \equiv \bar{a}_2 + \varepsilon_2$ and $b_{3r} \equiv \bar{a}_3 + \varepsilon_3$, respectively, where $\bar{a}_2 = E(a_2)$ and $\bar{a}_3 = E(a_3)$ and ε_2 and ε_3 are stochastic error terms with an expected value of zero. Inserting these terms in (7) and taking the logarithm, the econometric model can be written as:

$$(24) \quad \ln z_{\tau,r} = \ln B_r + b_{1r} \ln x_{\tau,1} + \bar{a}_{2r} \ln x_{\tau,2} + \bar{a}_{3r} \ln x_{\tau,3} + \varepsilon_{\tau,2} \ln x_{\tau,2} + \varepsilon_{\tau,3} \ln x_{\tau,3} \\ (\tau=1, \dots, 12; \quad \tau \in T_r)$$

where the index τ refers to observations within each recorded state, and where therefore $z_{\tau,r}$ is the τ 'th observation of output in state r . T_r is the total number of observations of state r in the sample.

The model involves two error terms (ε_2 and ε_3). If we define:

$$(25) \quad \varepsilon_\tau \equiv \varepsilon_{2\tau} \ln x_{\tau,2} + \varepsilon_{3\tau} \ln x_{\tau,3}$$

then (24) can be restated as:

$$(26) \quad \ln z_{\tau,r} = \ln B_r + b_{1r} \ln x_{\tau,1} + \bar{a}_{2r} \ln x_{\tau,2} + \bar{a}_{3r} \ln x_{\tau,3} + \varepsilon_\tau$$

where ε_τ is the error term with an expected value zero, and variance σ_ε^2 :

$$(27) \quad \sigma_\varepsilon^2 = (\ln(x_2))^2 \sigma_{a_2}^2 + (\ln(x_3))^2 \sigma_{a_3}^2$$

where $\sigma_{a_2}^2$ and $\sigma_{a_3}^2$ are the variances of state-variable 3 and 4, respectively, i.e., of the variance of the parameters a_2 and a_3 , respectively, according to the values and probabilities in Table 1 and 2. (Thus, the error term ε_τ in (8) is *heteroscedastic*).

Comparing (8) and (26) one sees that when estimating the model (8), the estimated values of b_{2r} and b_{3r} are in fact estimates of \bar{a}_{2r} (i.e. $E(a_2)$) and \bar{a}_{3r} (i.e. $E(a_3)$), respectively.

Appendix 3

Comparing the simulation setup (see (6)) and the estimation setup (see (11)), the parameters C , c_1 , c_2 , and c_3 may be expressed as $\ln C \equiv \overline{\ln A} + \varepsilon_{\ln A}$, $c_1 \equiv \bar{a}_1 + \varepsilon_1$, $c_2 \equiv \bar{a}_2 + \varepsilon_2$, and $c_3 \equiv \bar{a}_3 + \varepsilon_3$, respectively, where $\overline{\ln A} = E(\ln A)$, $\bar{a}_1 = E(a_1)$, $\bar{a}_2 = E(a_2)$ and $\bar{a}_3 = E(a_3)$, respectively, (according to values and probabilities in table 1 and 2), and where $\varepsilon_{\ln A}$, ε_1 , ε_2 and ε_3 are stochastic error terms with an expected value of zero. Defining the sum $\varepsilon \equiv \varepsilon_{\ln A} + \varepsilon_1 + \varepsilon_2 + \varepsilon_3$, then using these definitions and taking the logarithm, the model in (11) has the following econometric form:

$$(28) \quad \ln z_\tau = \overline{\ln A} + \bar{a}_1 \ln x_{1,\tau} + \bar{a}_2 \ln x_{2,\tau} + \bar{a}_3 \ln x_{3,\tau} + \varepsilon_\tau \quad (\tau=1, \dots, T)$$

where the index τ refers to observations and where ε_τ is the error term with an expected value zero, and the variance σ_ε^2 determined as:

$$(29) \quad \sigma_\varepsilon^2 = \sigma_{\ln A}^2 + (\ln(x_1))^2 \sigma_{a_1}^2 + (\ln(x_2))^2 \sigma_{a_2}^2 + (\ln(x_3))^2 \sigma_{a_3}^2$$

where $\sigma_{\ln A}^2$, $\sigma_{a_1}^2$, $\sigma_{a_2}^2$ and $\sigma_{a_3}^2$ are the variances of the logarithm of state variable 1, and the variances of state variables 2, 3 and 4, respectively, i.e., of the variance of the parameters $\ln A$, a_1 , a_2 and a_3 , respectively (see Table 1 and 2 in Section 4.2) Thus, the error term is *heteroscedastic*.

Comparing (12) and (28) it becomes clear, that when estimating the model (12) then the estimated values of C , c_1 , c_2 , and c_3 are in fact estimates of $\overline{\ln A}$ (i.e. $E(\ln A)$), \bar{a}_1 (i.e. $E(a_1)$), \bar{a}_2 (i.e. $E(a_2)$), and \bar{a}_3 (i.e. $E(a_3)$), respectively.

Appendix 4.

Estimation of variance

The variance of the error term in (12) is estimated as a function of input (following Judge et al., 1982, p. 417).

First the residuals \hat{e}_t were estimated as:

$$(30) \quad \hat{e}_t = z_t - \hat{z}_t$$

where \hat{z}_t is the predicted value estimated as:

$$(31) \quad \hat{z}_t = \hat{C} x_{1t}^{\hat{c}_1} x_{2t}^{\hat{c}_2} x_{3t}^{\hat{c}_3} \quad (t = 1, \dots, T)$$

where $\ln(\hat{C})$ and \hat{c}_k ($k=1,2,3$) are the estimated values of the parameters in (12).

Then, the following linear model was estimated using OLS:

$$(32) \quad \ln \hat{e}_t^2 = \alpha_1 + \alpha_2 \ln x_{1t} + \alpha_3 \ln x_{2t} + \alpha_4 \ln x_{3t} + v_t \quad (t=1, \dots, T)$$

An unbiased estimator of the variance of the error term \hat{e}_t in (30) is:

$$(33) \quad \hat{\sigma}^2 = \exp[(\hat{\alpha}_1 + 1.2704) + \hat{\alpha}_2 \ln x_1 + \hat{\alpha}_3 \ln x_2 + \hat{\alpha}_4 \ln x_3]$$

where $\hat{\alpha}_1$, $\hat{\alpha}_2$, $\hat{\alpha}_3$, and $\hat{\alpha}_4$ are the parameters estimated in (32).

As the net income in year t is $q_t = p_t z_t - (w_1 x_1 + w_2 x_2 + w_3 x_3) - c^F + k_t$, the estimated variance of the net income q is:

$$(34) \quad \hat{V}(q) = p^2 \hat{\sigma}^2$$

which concludes estimation of the variance.

Appendix 5.

Table A3. Estimated parameters of stochastic production function. Ordinary Least Squares. Average of two hundred simulations

Scenario	Average, units				Standard deviation			
Years-plots	C	c ₁	c ₂	c ₃	C	c ₁	c ₂	c ₃
50 - 1	10.384	0.174	0.224	0.237	24.762	0.191	0.201	0.155
50 - 2	3.672	0.149	0.211	0.240	0.434	0.016	0.024	0.011
50 - 3	3.654	0.149	0.212	0.240	0.327	0.013	0.021	0.008
100 - 1	11.750	0.171	0.248	0.249	35.759	0.183	0.186	0.158
100 - 2	3.699	0.157	0.261	0.243	0.367	0.012	0.019	0.011
100 - 3	3.690	0.156	0.262	0.243	0.265	0.010	0.015	0.008
200 - 1	6.033	0.158	0.246	0.269	6.930	0.117	0.120	0.095
200 - 2	3.705	0.156	0.260	0.261	0.246	0.009	0.013	0.007
200 - 3	3.694	0.156	0.260	0.262	0.180	0.007	0.011	0.005
400 - 1	4.906	0.161	0.265	0.287	3.549	0.088	0.095	0.072
400 - 2	3.729	0.158	0.269	0.291	0.193	0.006	0.009	0.006
400 - 3	3.736	0.157	0.269	0.291	0.137	0.005	0.007	0.005

Table A4. Estimated parameters of stochastic production function. Weighted Least Squares. Average of two hundred simulations

Scenario	Average, units				Standard deviation			
Years-plots	C	c ₁	c ₂	c ₃	C	c ₁	c ₂	c ₃
50 - 1	4.598	0.150	0.208	0.242	3.843	0.085	0.101	0.086
50 - 2	3.708	0.146	0.209	0.241	0.394	0.015	0.021	0.011
50 - 3	3.708	0.147	0.209	0.240	0.302	0.012	0.020	0.007
100 - 1	4.366	0.163	0.256	0.245	2.713	0.072	0.092	0.076
100 - 2	3.738	0.154	0.261	0.243	0.302	0.011	0.016	0.009
100 - 3	3.728	0.155	0.260	0.243	0.229	0.008	0.014	0.006
200 - 1	3.896	0.162	0.256	0.264	1.558	0.046	0.060	0.050
200 - 2	3.707	0.155	0.258	0.262	0.202	0.007	0.011	0.006
200 - 3	3.698	0.155	0.258	0.262	0.152	0.005	0.010	0.004
400 - 1	3.820	0.162	0.269	0.291	1.221	0.037	0.046	0.040
400 - 2	3.734	0.157	0.268	0.291	0.144	0.006	0.007	0.006
400 - 3	3.726	0.157	0.268	0.291	0.115	0.004	0.007	0.005

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